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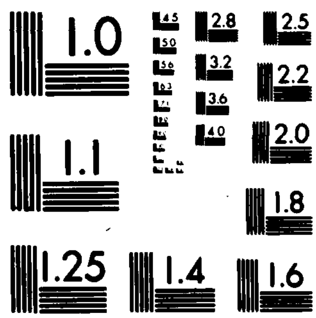


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The p-version of the finite element method is a new approach to finite element analysis which has been demonstrated to lead to significant computational savings, often by orders of magnitude (This approach was formerly called the constraint method; the new term p-version is more descriptive). Conventional approaches (called the h-version) generally employ low order polynomials as basis functions. Accuracy is achieved by suitably refining the approximating mesh. The p-version uses polynomials of arbitrary order $p \geq 2$ for problems in plane elasticity where C0 continuity is required and polynomials of order

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CONTENTS

<u>Discussion</u>

The p-version of the finite element method is a new approach to finite element analysis which has been demonstrated to lead to significant computational savings, often by orders of magnitude (This approach was formerly called the constraint method; the new term p-version is more descriptive). Conventional approaches (called the h-version) generally employ low order polynomials as basis functions. Accuracy is achieved by suitably refining the approximating mesh. The p-version uses polynomials of arbitrary order $p \geq 2$ for problems in plane elasticity where $C^0$ continuity is required and polynomials of order $p \geq 5$ for problems in plate bending where $C^1$ continuity is required.

Hierarchic elements which implement the p-version efficiently are used together with precomputed arrays of elemental stiffness matrices.

A major result was obtained in the previous reporting period on the convergence of the p-version of the finite element method <u>for the case of Co continuity</u>: let N be the number of degrees of freedom. In polynomial regions, if the h-version converges with order $0(1/N^{\alpha})$ then the p-version converges with order $0(1/N^{2\alpha})$ provided that all vertices of the polygon are vertices of the triangulation.

The theoretical order of convergence of the p-version in the case of $C^1$ continuity has still not been determined mathematically, although empirical results indicate that it is much more rapid than that of h-convergence. The basic problem of p-convergence <u>in the $C^1$ case</u> was investigated during this reporting period. By using modified Bernstein polynomials defined over triangles, it was shown that the p-version does indeed converge in the energy norm, although the optimal rate of convergence is unknown. The construction of these modified Bernstein polynomials, of interest in its own right, is presented in [1].

In the case of finite elements with curved boundaries, the standard h-version procedure is to use isoparametric mappings together with numerical quadrature to compute the stiffness matrix. In the p-version large elements are used and numerical quadrature becomes inefficient and expensive. Instead specific mappings are sought, which map curved triangles into the standard triangle, in such a way that the resulting elemental stiffness matrix can be computed using closed form integrals and pre-computed arrays. Such mappings have been developed in [2] for triangles with one curved side which is either elliptical or parabolic. The computer implementation of these elements will greatly increase the capabilities of the p-version for stress analysis.

Development of a hierarchic family of arbitrary polynomial order for a square-pyramidal element is now in progress.

Abstracts of Papers Presented at Meetings

[1] Smooth Approximation to a Function in $H_0^2(D)$ by Modified Bernstein Polynomials Over Triangles by A. G. Kassos and I. N. Katz, presented at the SIAM Fall 1979 Meeting, November 12-14, in Denver, Colorado.

Let f be defined on a bounded polygonal domain D in $R^2$ and let $S=\{T_i\}$, $i=1,\ldots,N$ be a triangulation of D. Using triangular coordinates, a Bernstein polynomial approximation to f of degree $p \geq 0$ can be constructed over each $T_i$. Denote this approximation by $B(f,p,T_i)$. It is easily shown that the global approximation induced by $B(f,p,T_i)$ $i=1,\ldots,N$ is continuous on D. It is, however, not smooth on D. In this paper, we construct a modified Bernstein polynomial approximation on each $T_i$, denoted by $B^*(f,p,T_i)$, for $p \geq 5$, when $f \epsilon C^1(D)$ and we show that the global approximation induced by $B^*(f,p,T_i)$ $i=1,\ldots,N$ is smooth. This result can be used to prove that any $f \epsilon H_0^2(D)$ can be approximated smoothly by a function which is a piecewise polynomial of sufficiently high degree p over a fixed triangulation S. The procedure in which S is fixed while p is increased is called the p-version of the finite element method, and its computer implementation has been described elsewhere.

[2] Triangles with One Curved Side for the P-version of the Finite Element Method, by I. N. Katz, presented at the SIAM Spring 1980 Meeting, June 5-7, in Alexandria Virginia.

In the p-version of the finite element method, the triangulation of the domain is fixed and the degree p of the approximating polynomial is varied. In this approach the triangles are generally large and numerical quadrature, in the case of curved elements, may be expensive. In order to circumvent this difficulty we consider the triangles with one curved side in which a particular shape is specified for the curved side (e.g. circular, parabolic, elliptic). A mapping of the curved element into a standard element is sought with the property that the resulting elemental stiffness matrix can be integrated in closed form (without recourse to numerical quadrature). Such mappings are given for the following two specifications of the curved and straight sides: (1) curved side parabolic, straight sides arbitrary (2) curved side elliptic, straight sides parallel to the axes of the ellipse.

## Professional Personnel

1. I. Norman Katz, Professor of Applied Mathematics and Systems Science, Washington University.

2. Barna A. Szabo, A. P. Greensfelder Professor of Civil Engineering, Washington University.

3. Douglas Wang, Research Assistant, doctoral candidate in the Department of Systems Science and Mathematics, Washington University.

1. PAPERS PUBLISHED AND PRESENTED SINCE THE START OF THE PROJECT (1977)

Published Papers:


1. "Hierarchal Finite Elements and Precomputed Arrays", by Mark P. Rossow and I. Norman Katz, Int. J. for Num. Method in Engr., Vol. 13, No. 6 (1978) pp. 977-999.

2. "Nodal Variables for Complete Conforming Finite Elements of Arbitrary Polynomial Order", by I. Norman Katz, A.G. Peano, and Mark P. Rossow, Computers and Mathematics with Applications , Vol 4, No. 2, (1978). pp. 85-112.

3. "A Hierarchic Family of Complete, Conforming $C^1$ Triangular Elements, for Plate Bending", by I. Norman Katz and Barna Szabo, (in preparation).

4. "Hierarchic Families of Complete Conforming Solid Finite Elements of Various Shapes", by I. Norman Katz, (in preparation).

5. "P-convergent Finite Element Approximations in Linear Elastic Fracture Mechanics", by Anil K. Mehta (doctoral dissertation), Department of Civil Engineering, Washington University (1978).

6. "An Improved p-version finite element algorithm and a convergence result for the p-version" by Anthony G. Kassos, Jr. (doctoral dissertation) Department of Systems Science and Mathematics, Washington University, (August, 1979).

7. "Hierarchic Families for the p-version of the finite element method", I. Babuska, I. N. Katz and B. A. Szabo, invited paper presented at the Third IMACS International Symposium on Computer Methods for Partial Differential Equations, published in Advances in Computer Methods for Partial Differential Equations - III (1979) pp. 278-286.

8. "The p-version of the Finite Element Method", I. Babuska, B.A. Szabo, and I.N. Katz, Technical Report No. WU/CCM-79/1, Center for Computational Mechanics, Washington University, St. Louis, MO (accepted for publication in SIAM Journal of Numerical Analysis).


Presented Papers:


9. "Hierarchical Approximation in Finite Element Analysis", by I. Norman Katz, International Symposium on Innovative Numerical Analysis in Applied Engineering Science, Versailles, France, May 23-27, 1977.

10. "Efficient Generation of Hierarchal Finite Elements Through the Use of Precomputed Arrays", by M.P. Rossow and I.N. Katz, Second Annual ASCE Engineering Mechanics Division Speciality Conference, North Carolina State University, Raleigh, NC May 23-25, 1977.

11. "$C^1$ Triangular Elements of Arbitray Polynomial Order Containing Corrective Rational Functions", by I. Norman Katz, SIAM 1977 National Meeting, Philadelphia, PA, June 13-15, 1977.

12. "Hierarchical Complete Conforming Tetrahedral Elements of Arbitrary Polynomial Order", by I. Norman Katz, presented at SIAM 1977 Fall Meeting, Albuquerque, NM, October 31- November 2, 1977.

13. "A Hierarchical Family of Complete Conforming Prismatic Finite Elements of Arbitrary Polynomial Order", by I. Norman Katz, presented at SIAM 1978 National Meeting, Madison, WI, May 24-26, 1978.

14. "Comparative Rates of h- and p- Convergence in the Finite Element Analysis of a Model Bar Problem", by I. Norman Katz, presented at the SIAM 1978 Fall Meeting, Knoxville Tennessee, October 20- November 1, 1978.

15. "Smooth Approximation to a function in $H_0^2(D)$ by modified Bernstein Polynomials over Triangles" by A. G. Kassos, Jr. and I. N. Katz, presented at the SIAM 1979 Fall Meeting, Denver, Colorado, November 12-14, 1979.

16. "Triangles with one curved side for the p-version of the finite element method" by I. Norman Katz, presented at the SIAM 1980 Spring Meeting, Alexandra VA, June 5-7, 1980.